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Propagation Characteristics of Hermite-Cosh-Gaussian Laser Beam In Relativistic Cold Quantum Plasma and Collisionless Plasma

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Abstract

In the present work, the authors have investigated the self-focusing and defocusing of Hermite-cosh-Gaussian laser (HChG) beam in relativistic cold quantum and collisionless plasmas. By using WKB and paraxial ray approximation for (TE_{00}) mode into account, an equation for envelope is set up and solved using Wentzel–Kramers–Brillouin and the paraxial ray approximation. An ordinary non-linear differential equation governing the beam width parameter as a function of propagation distance is set up for (TE_{00}) mode structures of the beam. Further, a numerical study of this differential equation is carried for suitable set of plasma and laser parameters. The beam undergoes self-focusing/defocusing due to non-linearity. Also the comparison between self-focusing/defocusing of HChG beam in the relativistic cold quantum plasma and collisionless plasmas. In presence of collisionless plasma not only leads to substantial increase in self-focusing length, but also results in oscillatory character with decreasing f. While in relativistic cold quantum plasma, strong self-focusing and defocusing is observed. Further, self-focusing is enhanced with increased value of decentred parameter.

Keywords: Non-linear Dynamics, Beam width parameter, Hermite-cosh-Gaussian beam, Relativistic cold quantum plasma, Collisionless plasma Self-focusing and defocusing.

Introduction

The earlier investigation on self-focusing has been confined to cylindrical symmetric Gaussian laser beam. The propagation of Gaussian beam, Hermite–Gaussian beam, cosh-Gaussian beam, Hermite-Cosh-Gaussian (HChG) beam, etc. are recently studied by researchers due to its wide ranging applications. [1-10].

The numerically investigate the effect of non-linearity and decentred parameter on the propagation of Hermite-Cosh-Gaussian beams in collisionless plasma. The decentred parameter plays a crucial role to enhance the self-focusing of HChG beams in a non-linear medium. The relativistic self-focusing of Cosh-Gaussian beam through dense plasma of density ramp profile by using higher-order paraxial approximation [11, 12].

Non-linear dynamics for relativistic cold quantum plasma and collisionless plasma:

The electric field distribution of Hermite-Cosh-Gaussian (HChG) laser beams propagating in plasma along z-axis is of the following form:

$$E(x,y,z) = \frac{E_0}{\sqrt{f_1(z) f_2(z)}} \mathrm{H}\left(\frac{\sqrt{2}x}{f_1(z) r_0}\right) \mathrm{H}\left(\frac{\sqrt{2}y}{f_2(z) r_0}\right) \times e^{-\left(\frac{x^2}{f_1^2(z) r_0^2} + \frac{y^2}{f_2^2(z) r_0^2}\right)} \times \cos\left(\frac{\Omega_0 x}{f_1(z) r_0}\right) \cos\left(\frac{\Omega_0 y}{f_2(z) r_0}\right) \quad (1)$$

Where, H is Hermite polynomial, E_0 is amplitude of electric field for HChG laser beam, r_0 is the waist width, $f_1(z)$ and $f_2(z)$ are the dimensionless beam width parameters along x and y directions respectively, b is the decentred parameter, Ω_0 is the parameter associated with the cosine function.

The dielectric constant for the non-linear medium is of the following form [13]

$$\varepsilon = \varepsilon_0 + \emptyset\,(EE^*) \quad (2)$$

Here,

ε_0 and $\emptyset$ represent the linear and nonlinear part of dielectric constant respectively, and $\varepsilon_0 = \left(1 - \frac{\omega_p^2}{\omega_0^2}\right)$ with ω as the incident laser frequency and ω_p is the plasma frequency , The form of the function $\emptyset$ is different in different physical situations, but the dependence on EE^* is a common feature. With increasing beam power dielectric constant tends to saturation value.

Now consider the dielectric constant for relativistic cold quantum plasma by y Jung and Murakami [14]

$$\varepsilon = 1 - \frac{\omega_p^2}{\gamma\omega^2}\left(1 - \frac{\delta q}{\gamma}\right)^{-1} \quad (3)$$

Where,

$\delta q = \frac{4\pi^2 h^2}{m^2 \omega^2 \lambda^4}$, h is Planck constant and $\gamma = \sqrt{1 + \alpha EE^*}$ is relativistic factor,

$\alpha = \frac{e^2}{m^2 \omega^2 c^2}$; Where c is speed of light in the vacuum.

The WKB approximation assuming that the variations in the z direction are slower than that in radial direction,

$$-2ik\frac{\partial A}{\partial z} + \nabla^2 + \frac{\omega^2}{c^2}\emptyset(EE^*)A = 0 \qquad (4)$$

The real part of above equation is,

$$2\left(\frac{\partial s}{\partial z}\right) + \left(\frac{\partial s}{\partial x}\right)^2 + \left(\frac{\partial s}{\partial y}\right)^2 = \frac{1}{k^2 A_0}\left(\frac{\partial^2 A_0}{\partial x^2} + \frac{\partial^2 A_0}{\partial y^2}\right) + \frac{\emptyset(A_0^2)}{\varepsilon_0} \qquad (5)$$

And the imaginary part is,

$$\frac{\partial A^2}{\partial z} + \frac{\partial s}{\partial x}\frac{\partial A_0^2}{\partial x} + \frac{\partial s}{\partial y}\frac{\partial A_0^2}{\partial y} + A^2\left(\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2}\right) = 0 \qquad (6)$$

The solution of above equation is

$$2\left(\frac{\partial s}{\partial z}\right) + \left(\frac{\partial s}{\partial x}\right)^2 + \left(\frac{\partial s}{\partial y}\right)^2 = \beta_1(z)x^2 + \beta_2(z)y^2 \qquad (7)$$

Here,

$$\beta_1(z) = \frac{1}{f_1(z)}\frac{df_1}{dz}$$

$$\beta_2(z) = \frac{1}{f_2(z)}\frac{df_2}{dz}$$

Now, the electric field distribution of Hermite-Cosh-Gaussian (HChG) laser beams propagating in plasma along z-axis is of the following form:

$$E(x,y,z) = \frac{E_0}{\sqrt{f_1(z)f_2(z)}} H\left(\frac{\sqrt{2}x}{f_1(z)r_0}\right) H\left(\frac{\sqrt{2}y}{f_2(z)r_0}\right) \times e^{-\left(\frac{x^2}{f_1^2(z)r_0^2} + \frac{y^2}{f_2^2(z)r_0^2}\right)} \times \cos\left(\frac{\Omega_0 x}{f_1(z)r_0}\right)\cos\left(\frac{\Omega_0 y}{f_2(z)r_0}\right) \qquad (8)$$

Where, H is Hermite polynomial, E_0 is amplitude of electric field for HChG laser beam, r_0 is the waist width, $f_1(z)$ and $f_2(z)$ are the dimensionless beam width parameters along x and y directions respectively, b is the decentred parameter, Ω_0 is the parameter associated with the cosine function.

The dielectric constant for the non-linear medium (collisionless plasma) is of the following form [13]

$$\varepsilon = \varepsilon_0 + \emptyset\,(EE^*) \qquad (9)$$

Here ε_0 and $\emptyset$ represent the linear and nonlinear part of dielectric constant respectively.

The non-linearity induced due to ponderomotive forces in the dielectric constant in collisionless plasma and hence non-linear part of the dielectric constant can be written as [15]

$$\phi(EE^*) = \left(\frac{\omega_p^2}{\omega_0^2}\right)\left[1 - exp\left(\frac{3m\alpha(EE^*)}{4M}\right)\right]$$

Where, ω_p is the plasma frequency, ω_0 is the frequency of laser beam.

Beam width parameter differential equations for HChG beam in relativistic cold quantum plasma:

Now employing the beam width paraxial approximation expression for $f_1(z)\, and\, f_2(z)$ as:

$$\frac{\partial^2 f_1(z)}{\partial \xi^2} = \frac{4}{k^2 f_2^4 r_0^4} - \frac{4b^2}{k^2 f_2^4 r_0^4} - \frac{2}{f_1 f_2^3 R_n^2} + \frac{b^2}{f_1 f_2^3 R_n^2} \quad (10)$$

$$\frac{\partial^2 f_2(z)}{\partial \xi^2} = \frac{4}{k^2 f_1^4 r_0^4} - \frac{4b^2}{k^2 f_1^4 r_0^4} - \frac{2}{f_2 f_1^3 R_n^2} + \frac{b^2}{f_2 f_1^3 R_n^2} \quad (11)$$

Where b is decentred parameter and ξ is dimensionless distance of propagation.
The above equations are the expression for the beam width parameters f_1 and f_2 for $\mathbf{TE}_{00}$ mode respectively.

Beam width parameter differential equation for HChG beam in collisionless plasma:

$$\frac{\partial^2 f_1(z)}{\partial \xi^2} = -\frac{4(-1+b^2)}{f_1^3(z)} + \frac{3(-2+b^2)e^{\frac{-3mp}{4Mf_1f_2}} p\, m r_0^2 \omega_p^2}{4c^2 M f_1^3(z) f_2^2(z)} \quad (12)$$

$$\frac{\partial^2 f_2(z)}{\partial \xi^2} == -\frac{4(-1+b^2)}{f_2^3(z)} + \frac{3(-2+b^2)e^{\frac{-3mp}{4Mf_1f_2}} p m r_0^2 \omega_p^2}{4c^2 M f_2^3(z) f_1^2(z)} \quad (13)$$

Self trapping condition:

For initially plane wave front, $\frac{\partial f_1(z)}{\partial \xi} = 0, f_{1(z)} = 0, \frac{\partial f_2(z)}{\partial \xi} = 0, f_{2(z)} = 0$, the condition $\frac{\partial^2 f_1(z)}{\partial \xi^2} = 0$, $\frac{\partial^2 f_2(z)}{\partial \xi^2} = 0$, leads to the propagation of laser beam in self –trapped mode. The critical conditions and their graphical representation are termed as the critical curve. To explain the results for propagation of HChG laser beam in plasma, we numerically analyze the dependence of dimensionless initial beam width ρ as function of p for collision less plasma under self-trapped condition.

Results and Discussions

We conduct the numerical analysis and computational simulations for solving the beam width parameter equations.

The results are shown in figure.1 (critical curve analysis for relativistic cold quantum plasma) below. It is found that decrease in ρ is observed with increase in p for plasma at different values of decentred parameter.

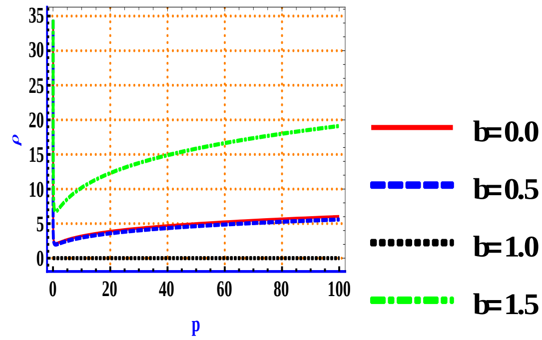

Figure1. Dependence of dimensionless beam width parameter as a function of power of plasma p and this is the critical curve analysis for relativistic cold quantum plasma.

Moreover, we conduct the numerical analysis and computational simulations for solving the beam width parameter equations.

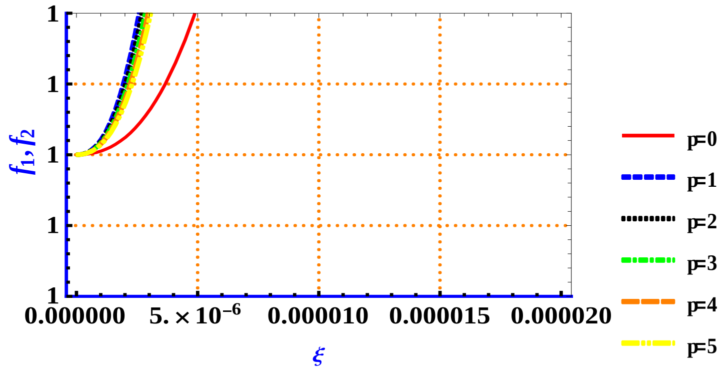

Figure2. The variation of beam width parameter *f* with normalized propagation distance ξ for different value of p when ρ= 2.1320503832612845 and the decentred parameter b=0.

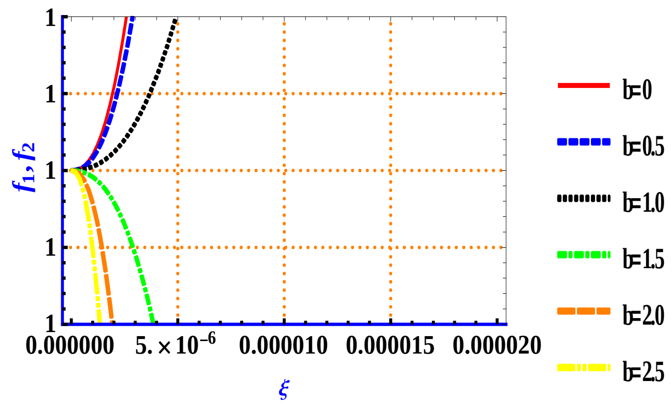

Figure3. The variation of beam width parameter *f* with the normalized propagation distance ξ with different values of decentred parameter b, when p= 0.9112888355269175 which is critical value of power of plasma.

Above figure shows that the self focusing of HChG beam for values of b (that is b=1.5, 2.0, 2.5) and the defocusing of HChG beam for the values of b=0.0, b=0.5 and b=1.0 takes place.

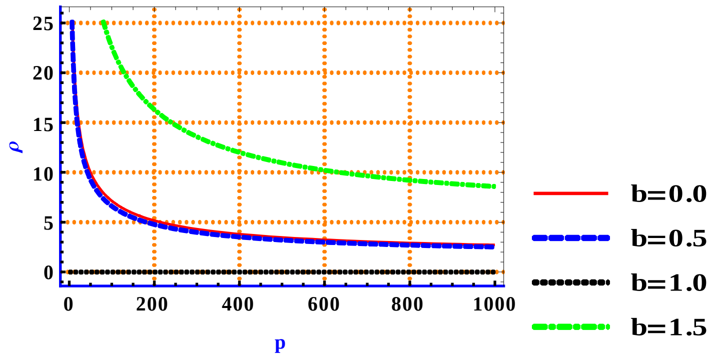

Figure 4. **Dependence of dimensionless beam width parameter as a function of intensity or power of plasma p and this is the critical curve analysis for collisionless plasma.**

Figure below shows that for different value of the decentred parameter b=0.50 and b=1.00 the self-focusing of HChG beam takes place and for the decentered parameter b=0 the self focusing of HChG beam takes place and the beam goes abruptly to infinity.

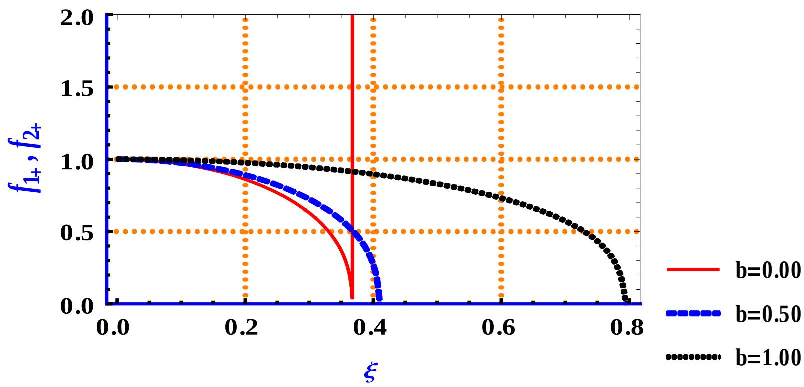

Figure 5.The variation of beam width parameter *f* with the normalized propagation distance ξ.

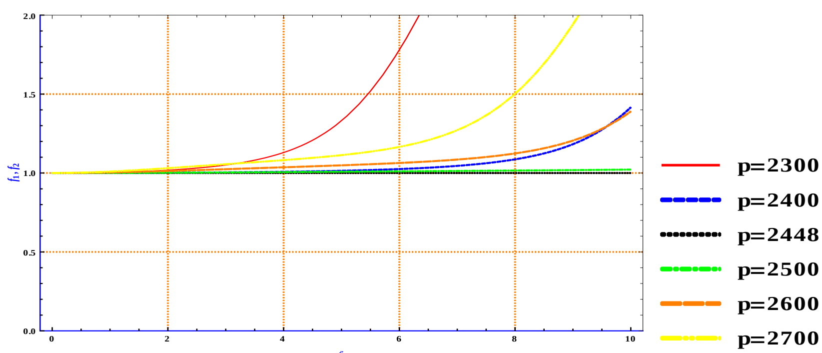

Figure6. The variation of beam width parameter *f* with normalized propagation distance ξ for different value of p when ρ=2 and the decentred parameter b=0.

In above figure 6 for the critical value of plasma power (p=2448) the self trapping takes place and for the values less than p and greater than p the defocusing of HChG beam takes place.

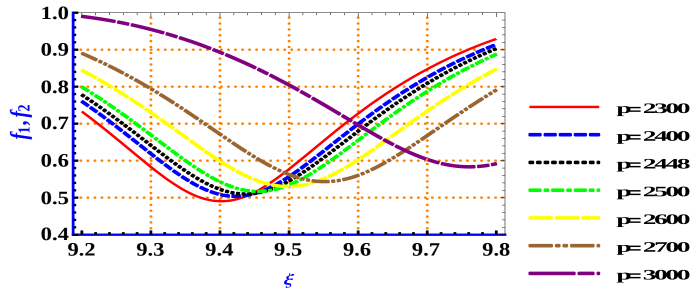

Figure7. The variation of beam width parameter *f* with normalized propagation distance ξ with different value of p for decentred parameter b=0 and ρ=3.

Above figure shows the plasma oscillations.

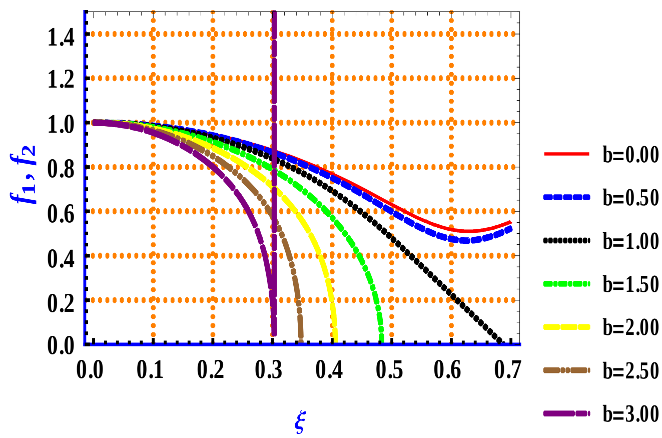

Figure8. The variation of beam width parameter *f* with the normalized propagation distance ξ with different values of decentred parameter b, when p=2448 which is critical value of power of plasma.

Above figure shows that the self focusing of HChG beam for values of b (that is b=1.0, 1.50, 2.00, 2.50).

For b=3.00 the HChG beam is self-focused and goes abruptly to infinity.

For b=0.50 and b=1.00 the plasma shows oscillatory behaviour.

Conclusion

From the above results, we conclude that the self-focusing and defocusing of HChG beam in relativistic cold quantum plasma for TE_{00} mode controlled with the plasma power and the different values of decentred parameter. Moreover, the higher values of decentred parameter that is for b=1.5, b=2.0, b=2.5 the self-focusing of beam occurs for lower values of decentred parameter that is for b=0, b=0.5, b=1.0 the defocusing of beam takes place. This indicates that decentred parameter play important role in self-focusing and defocusing of plasma. This implies that for the higher value of decentred parameter beam shows self-focusing. While in collisionless plasma for higher values of decentered parameter that is b=3, the self-focusing of the beam occurs and the beam abruptly goes to infinity. This indicates that decentred parameters play important role in self-focusing of HChG beam. The plot between beam width parameter *f* and the normalized length ξ for different value of decentred parameters and power of plasmas has been reported and indicates the enhancement of the self-focusing phenomenon of the laser beams in the plasmas.

Conflicts of interest: The authors stated that no conflicts of interest.

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